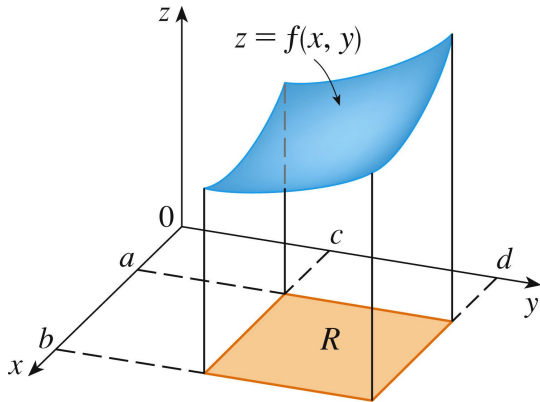


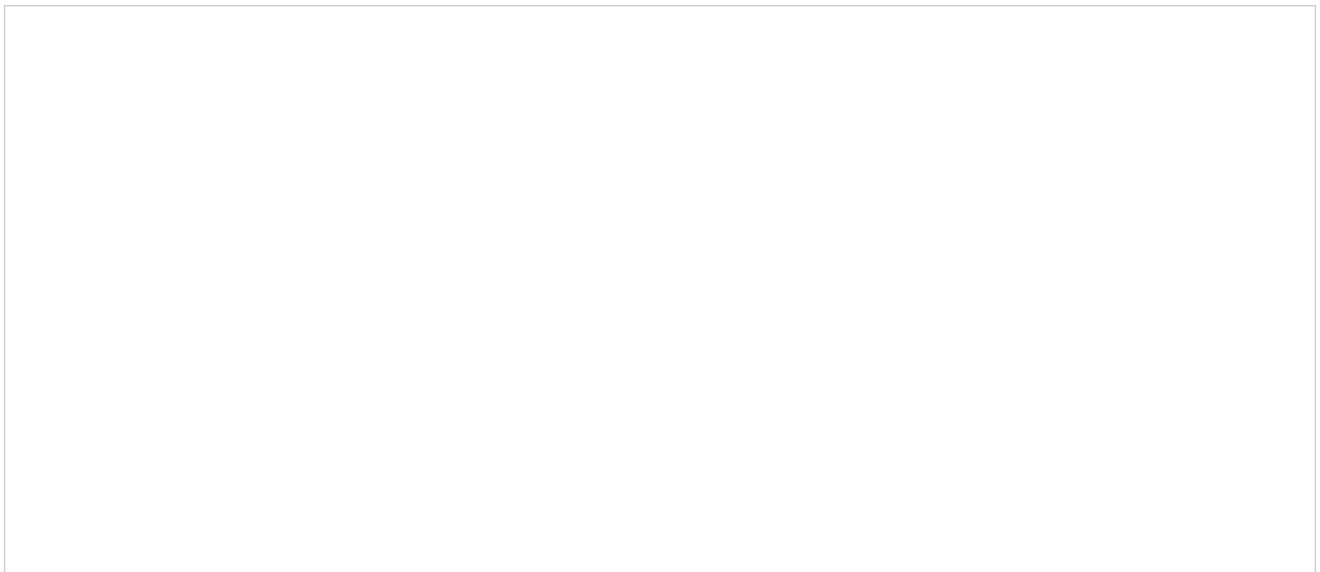
Lesson 23a. Double Integrals Over General Regions

0 Warm up – double integrals over rectangles

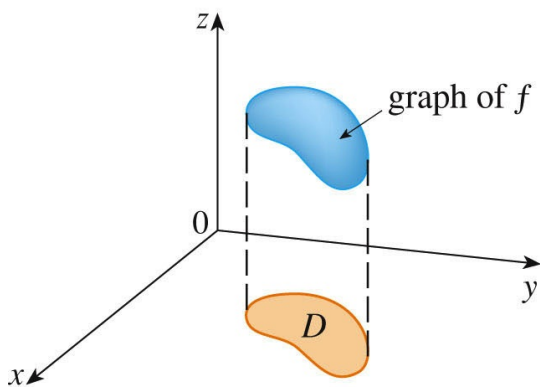


- Rectangle $R = [a, b] \times [c, d]$
 $= \{(x, y) \mid a \leq x \leq b, c \leq y \leq d\}$
- $\iint_R f(x, y) dA = \int_a^b \int_c^d f(x, y) dy dx$
 $= \int_c^d \int_a^b f(x, y) dx dy$

Example 1. Find $\int_0^1 \int_0^1 (x + y) dy dx$.



1 General regions

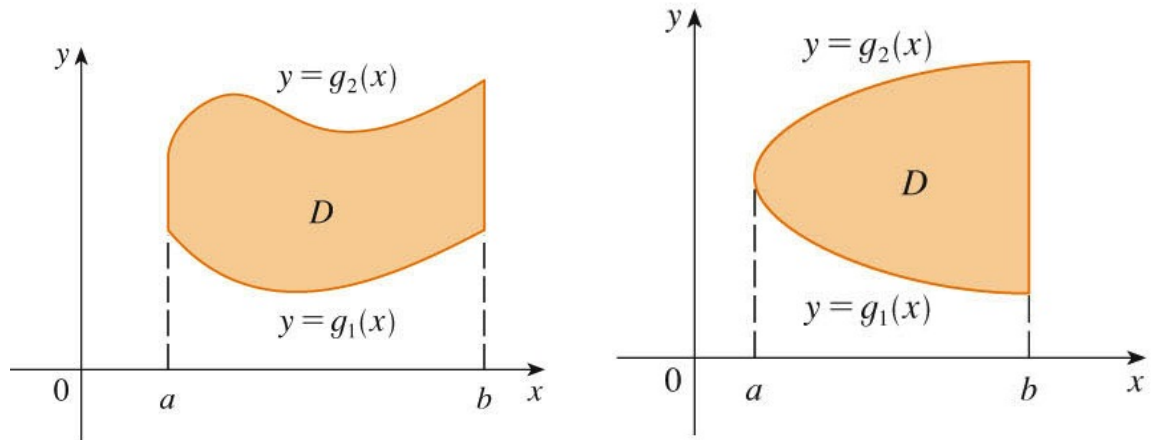


- How about general regions D ?
- Intuition: if $f(x, y) \geq 0$, double integral still represents volume of solid between D and graph of f
- We focus on two types of regions

- **Type I regions:** lies between two vertical lines and the graphs of two continuous functions of x , that is:

$$D = \{(x, y) \mid a \leq x \leq b, g_1(x) \leq y \leq g_2(x)\}$$

- Examples:



- If D is a type I region (and f is continuous on D), then

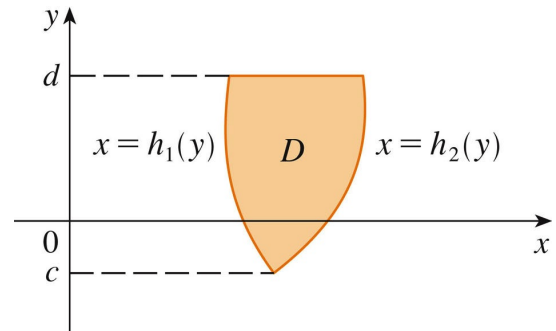
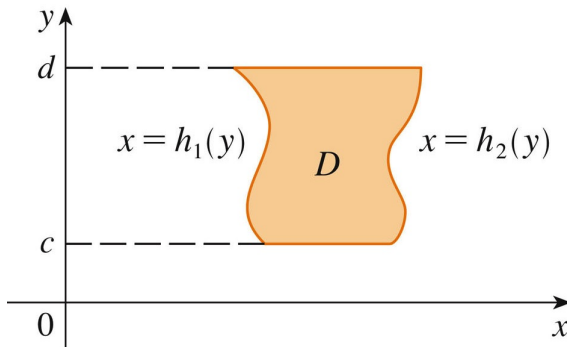
- In the inner integral, x is regarded a constant in $f(x, y)$ and the limits of integration

Example 2. Find $\iint_D (x + 3) dA$, where D is the region bounded by $y = 0$ and $y = x^2$, and $x = 2$.

- **Type II regions:** lies between two horizontal lines and the graphs of two continuous functions of y , that is:

$$D = \{(x, y) \mid c \leq y \leq d, h_1(y) \leq x \leq h_2(y)\}$$

- Examples:



- If D is a type II region (and f is continuous on D), then

- In the inner integral, y is regarded a constant in $f(x, y)$ and the limits of integration

Example 3. Find $\iint_D y \, dA$, where D is the triangular region with vertices $(0, 0)$, $(2, 0)$, $(1, 1)$.

2 If we have time...

Example 4. Find $\iint_D (x - y) dA$, where D is enclosed by the lines $y = x$, $x = 0$, and $y = 1$.

